Appendix A: Example Calculations of Statistical Methods

The following calculations are examples of the statistical procedures presented in Section 3.

- Example 1 includes concentration and flow results for 12 baseline and monitoring samples. This example presents application of both Method 1 and Method 2, using the steps defined for 12 samples.
- Example 2 includes loading results for 18 baseline and monitoring samples. This example presents application of both Method 1 and Method 2, and includes the calculation of the Single Observation Trigger specified for 17 samples or greater in Method 1.
- Example 3 includes concentration and flow results for 12 baseline and monitoring samples. This example presents application of both Method 1 and Method 2 and demonstrates a recommended approach when replacing baseline concentrations below the limits established in 40 CFR part 434, Subpart C.

1.0 Example 1

Assume 12 baseline flow and iron concentrations are collected by sampling once per month for a year. Likewise, 12 flow and iron monitoring observations are obtained by sampling once per month for a period of one year. Determination of exceedances are presented using both Methods 1 and 2. For all calculations in Example 1, assume the following flows (in gpm) and iron concentrations (in mg/L).

Flow

Baseline	5.0	14.0	42.0	35.0	26.0	22.0	12.0	11.0	11.0	6.0	11.0	6.0
Monitoring	8.0	14.0	15.0	32.0	43.0	28.0	16.0	14.0	16.0	9.0	9.0	11.0

Iron Concentration

Baseline	14.2	14.0	20.6	13.6	13.2	12.4	13.2	13.4	13.6	14.3	15.2	13.4
Monitoring	16.2	17.8	16.3	15.1	20.9	15.7	15.8	16.7	15.4	15.6	16.7	15.3

The resulting iron loads (in lbs/day = flow * concentration * 0.01202) are given below:

Iron Loads

Baseline	0.85	2.36	10.40	5.72	4.13	3.28	1.90	1.77	1.80	1.03	2.01	0.97
Monitoring	1.56	3.00	2.94	5.81	10.80	5.28	3.04	2.81	2.96	1.69	1.81	2.02

1.1 Single Observation Trigger:

1.1.1 Method 1 (See Figure 3.2a):

1) Twelve baseline observations were collected, therefore n = 12.

2) The baseline loading observations are placed in sequential order from smallest to largest. [0.85, 0.97, 1.03, 1.77, 1.80, 1.90, 2.01, 2.36, 3.28, 4.13, 5.72, 10.40]

3) The number of observations, n, is less than 16, therefore the Single Observation Trigger (L) equals $x_{(12)}$ (the maximum) = 10.40.

4) One monitoring load (10.80) is greater than 10.40, therefore the Single Observation Trigger (L) was exceeded.

1.1.2 Method 2 (See Figure 3.2b):

1) Twelve is an even number, therefore the median of the modified baseline observations is:
   $M = 0.5 * (x_{(6)} + x_{(7)})$.
   $M = 0.5 * (1.90 + 2.01) = 1.955$

   In order to determine M_1, calculate the median of the subset ranging from $x_{(7)}$ to $x_{(12)}$. Because 12 - 6 = 6 is even, $M_1 = 0.5 * (x_{(9)} + x_{(10)})$
   $M_1 = 0.5 * (3.28 + 4.13) = 3.705$

2) In order to determine M_{-1}, calculate the median of the subset ranging from $x_{(1)}$ to $x_{(6)}$. Because 6 is even, $M_{-1} = 0.5 * (x_{(3)} + x_{(4)})$
   $M_{-1} = 0.5 * (1.03 + 1.77) = 1.40$

3) To calculate R, subtract M_{-1} from M_1.
   R = 3.705 - 1.40 = 2.305

4) $L = M_1 + (3 * R) = 3.705 + (3 * 2.305) = 10.62$

5) One monitoring observation (10.80) is greater than 10.62, therefore the Single Observation Trigger (L) was exceeded.

1.2 Annual Comparison

1.2.1 Method 1 (See Figure 3.2a)

1) From 1.1.2 Step 1, $M_1 = 3.705$

2) From 1.1.2 Step 2, $M_{-1} = 1.40$

3) From 1.1.2 Step 3, R = 2.305

4) The calculated value for R is then substituted into the equation for T.

$$T = 1.955 + \frac{1.815 * 2.305}{\sqrt{12}} = 3.16$$

5) The following monitoring observations are ordered from smallest to largest.
[1.56, 1.69, 1.81, 2.02, 2.81, 2.94, 2.96, 3.00, 3.04, 5.28, 5.81, 10.80]

6) There are 12 monitoring observations, therefore m = 12.
The number of observations is even, therefore M' = 0.5 * ($x_{(6)}$ + $x_{(7)}$)
M' = 0.5 * (2.94 + 2.96) = 2.95
This holds true for M_1' and M_{-1}' as well.
M_1' = 0.5 * ($x_{(9)}$ + $x_{(10)}$) = 0.5 * (3.04 + 5.28) = 4.16
M_{-1}' = 0.5 * ($x_{(3)}$ + $x_{(4)}$) = 0.5 * (1.81 + 2.02) = 1.915

7) To calculate R, subtract M_{-1}' from M_1'
R' = 4.16 - 1.915 = 2.245.

8) The calculated value for R' is then substituted in the equation for T '.

$$T' = 2.95 - \frac{1.815 * 2.245}{\sqrt{12}} = 1.77$$

9) T' (1.77) is less than T (3.16), therefore the median baseline pollution loading was not exceeded.

1.2.2 Method 2 (Wilcoxon-Mann-Whitney Test) (See Figure 3.2b)

Instructions for the Wilcoxon-Mann-Whitney test are given in Conover (1980), cited in Figure 3.2b.

1) When using both baseline and monitoring data, n = 12 and m =12

2) The baseline and monitoring observations are listed with their corresponding rankings.

Baseline Observations (lbs/day)	0.85	0.97	1.03	1.77	1.80	1.90	2.01	2.36	3.28	4.13	5.72	10.40
Baseline Rankings	1	2	3	6	7	9	10	12	18	19	21	23
Monitoring Observations (lbs/day)	1.56	1.69	1.81	2.02	2.81	2.94	2.96	3.00	3.04	5.28	5.81	10.80
Monitoring Rankings	4	5	8	11	13	14	15	16	17	20	22	24

3) The sum of the twelve baseline ranks (S_n) = 131.

4) In order to find the appropriate critical value (C), match the column with the correct n (number of baseline observations) to the row with the correct m (number of monitoring observations). As found in the table, the critical value C for 12 baseline and 12 monitoring observations is 99.

Critical Values (C) of the Wilcoxon-Mann-Whitney Test
(for a one-sided test at the 99.9 percent level)

n m	10	11	12	13	14	15	16	17	18	19	20
10	66	79	93	109	125	142	160	179	199	220	243
11	68	82	96	112	128	145	164	183	204	225	248
12	70	84	99	115	131	149	168	188	209	231	253
13	73	87	102	118	135	153	172	192	214	236	259
14	75	89	104	121	138	157	176	197	218	241	265
15	77	91	107	124	142	161	180	201	223	246	270
16	79	94	110	127	145	164	185	206	228	251	276
17	81	96	113	130	149	168	189	211	233	257	281
18	83	99	116	134	152	172	193	215	238	262	287
19	85	101	119	137	156	176	197	220	243	268	293
20	88	104	121	140	160	180	202	224	248	273	299

5) S_n (131) is greater than C (99). Therefore, according to the Wilcoxon-Mann-Whitney Test, the monitoring observations did not exceed the baseline pollution loading.

2.0 Example 2

Assume 18 baseline iron loading determination observations are collected by sampling twice per month for nine months. Likewise, 18 iron load monitoring observations are obtained by sampling twice per month for a period of nine months. Examples of both Methods 1 and 2 are presented below. For all calculations in Example 2, assume the following iron load observations (in lbs/day):

Observation	Baseline	Monitoring
1	0.030	0.530
2	0.005	0.040
3	1.915	1.040
4	0.673	0.033
5	0.064	0.030
6	0.063	0.230
7	0.607	0.710
8	0.553	0.240
9	0.286	0.390
10	0.106	0.830
11	0.406	3.050
12	1.447	0.580
13	0.900	1.180
14	0.040	0.510
15	2.770	0.046
16	1.803	0.690
17	0.160	0.630
18	0.045	0.370

<u>2.1 Single Observation Trigger</u>

2.1.1 Method 1 (See Figure 3.2a)

1) The number of baseline observations collected, n = 18.

2) The baseline observations are ordered sequentially from smallest to largest. [0.005, 0.030, 0.040, 0.045, 0.063, 0.064, 0.106, 0.160, 0.286, 0.406, 0.553, 0.607, 0.673, 0.900, 1.447, 1.803, 1.915, 2.770]

3) The number of observations is greater than 16, therefore M, M_1, M_2 and M_3 must be calculated. The number of observations is even, which means the median of the baseline observations must be calculated using the following equation:
$M = 0.5 * (x_{(9)} + x_{(10)})$.
$M = 0.5 * (0.286 + 0.406) = 0.346$

4) To determine M_1, calculate the median of the subset ranging from $x_{(10)}$ to $x_{(18)}$.
18 - 9 = 9 is odd, therefore $M_1 = x_{(14)} = 0.900$.

5) To determine M_2, calculate the median of the subset ranging from $x_{(14)}$ to $x_{(18)}$.
18 - 13 = 5 is odd, therefore $M_2 = x_{(16)} = 1.803$.

6) To determine M_3, calculate the median of the subset ranging from $x_{(16)}$ to $x_{(18)}$.
18 - 15 = 3, which is odd, therefore $M_3 = x_{(17)} = 1.915$.

7) To determine L, calculate the median of the subset ranging from $x_{(17)}$ to $x_{(18)}$.
18 - 16 = 2, which is even, therefore $L = 0.5 * (x_{(17)} + x_{(18)}) = 0.5 * (1.915 + 2.770) = 2.343$.

8) One monitoring observation (3.050) is above L (2.343), therefore the Single Observation Trigger was exceeded.

2.1.2 Method 2 (See Figure 3.2b)

1) From 2.1.1 Step 4, M_1 (the third quartile of the baseline data) is equal to 0.900.

2) To find M_{-1}, calculate the median of the subset ranging from $x_{(1)}$ to $x_{(9)}$.
9 is odd, therefore $M_{-1} = x_{(5)} = 0.063$.

3) The value for R is found by subtracting M_{-1} from M_1
$R = 0.900-0.063 = 0.837$

4) $L = M_1 + (3 * R) = 0.900 + (3 * 0.837) = 3.411$.

5) All monitoring observations are less than 3.411, therefore the Single Observation Trigger (L) was not exceeded.

2.2 Annual Comparison

2.2.1 Method 1 (See Figure 3.2a)

1) As determined in Section 2.1.2 step 1, M = 0.346, and M_1 = 0.900.

2) As determined in Section 2.1.2 step 2, M_{-1} = 0.063.

3) As determined in Section 2.1.2 step 3, R = 0.837.

4) To find T, the value for R is inserted in the following equation:

$$T = 0.346 + \frac{1.815 * 0.837}{\sqrt{18}} = 0.704$$

5) The monitoring observations are placed in order from lowest to highest.
[0.030, 0.033, 0.040, 0.046, 0.230, 0.240, 0.370, 0.390, 0.510, 0.530, 0.580, 0.630, 0.690, 0.710, 0.830, 1.040, 1.180, 3.050]

6) The number of monitoring observations (m) = 18.
18 is even, making M' = 0.5 * ($x_{(9)} + x_{(10)}$)
M' = 0.5 * (0.510 + 0.530) = 0.520

7) To determine M_1', calculate the median of subset $x_{(10)}$ to $x_{(18)}$.
Because 18 - 9 = 9 is odd, M_1' = ($x_{(14)}$) = 0.710

8) To determine M_{-1}', calculate the median of subset $x_{(1)}$ to $x_{(9)}$.
Because 9 is odd, M_{-1}' = ($x_{(5)}$) = 0.230

9) The value for R' is found by subtracting M_{-1}' from M_1'.
R' = 0.710 - 0.230 = 0.48

10) To find T ', the value for R' is inserted into the following equation:

$$T' = 0.520 - \frac{1.815 * 0.48}{\sqrt{18}} = 0.315$$

11) T ' (0.315) is less than T (0.704), therefore the median baseline pollution loading is not exceeded.

2.2.2 Method 2 (Wilcoxon-Mann-Whitney Test) (See Figure 3.2b)

Instructions for the Wilcoxon-Mann-Whitney test are given in Conover (1980), cited in Figure 3.2b.

1) When using both baseline and monitoring data, n = 18 and m = 18.

2) The baseline and monitoring observations are listed in order of collection, and ranked as follows:

Baseline Observations		Monitoring Observations	
(lbs/day)	(Ranking)	(lbs/day)	(Ranking)
0.030	2.5	0.530	20
0.005	1	0.040	6
1.915	34	1.040	30
0.673	25	0.033	4
0.064	10	0.030	2.5
0.063	9	0.230	13
0.607	23	0.710	27
0.553	21	0.240	14
0.286	15	0.390	17
0.106	11	0.830	28
0.406	18	3.050	36
1.447	32	0.580	22
0.900	29	1.180	31
0.040	5	0.510	19
2.770	35	0.046	8
1.803	33	0.690	26
0.160	12	0.630	24
0.045	7	0.370	16

The value of 0.030 was obtained for more than one observation. The ranking displayed is the average of 2 and 3 (2.5).

3) The sum of the 18 baseline ranks (S_n) = 322.5.

4) From the table in section 1.2.2 of this appendix, the critical value (C) for 18 baseline and

18 monitoring observations is 238.

5) S_n (322.5) is greater than the critical value C (238). Therefore, according to the Wilcoxon-Mann-Whitney test, the monitoring observations did not exceed the baseline pollution loading.

3.0 Example 3

Assume 12 baseline flow and iron concentrations are collected by sampling once per month for a year. Likewise, 12 flow and iron monitoring observations are obtained by sampling once per month for a period of one year. In order to determine whether baseline pollution loading has been exceeded, both Methods 1 and 2 were used. For all calculations in Example 3, assume the following flows (in gpm) and iron concentrations (in mg/L).

Flow

Baseline	5.0	12.0	15.0	34.0	21.0	11.0	16.0	9.0	10.0	11.0	9.0	13.0
Monitoring	7.0	11.0	17.0	29.0	22.0	12.0	13.0	14.0	10.0	12.0	11.0	9.0

Iron Concentration

Baseline	11.4	8.2	6.0	11.1	6.4	10.3	12.1	14.2	6.1	8.3	10.0	13.5
Monitoring	12.3	13.5	9.8	7.9	5.8	7.5	8.2	9.3	8.4	12.5	14.1	15.3

Because there are three baseline concentrations (6.0 mg/L, 6.4 mg/L and 6.1 mg/L) below the Subpart C effluent limit for iron (7.0 mg/L), two separate sets of loading results are calculated. The first calculates iron loading using all the unmodified concentrations, and the following standard equation:

Load (in lbs/day) = Flow (in gpm) * Concentration (in mg/L) * 0.01202.

The resulting iron loads are given below:

Iron Load (using unmodified concentrations)

Baseline	0.69	1.18	1.08	4.54	1.62	1.36	2.33	1.54	0.73	1.10	1.08	2.11
Monitoring	1.03	1.78	2.00	2.75	1.53	1.08	1.28	1.57	1.01	1.80	1.86	1.66

The second set of calculated iron loads are calculated after replacing the baseline iron concentration below 7.0 mg/L with 7.0 mg/L. This set is given below, with the three modified loads in bold:

Iron Load (using modified concentrations)

Baseline	0.69	1.18	**1.26**	4.54	**1.76**	1.36	2.33	1.54	**0.84**	1.10	1.08	2.11
Monitoring	1.03	1.78	2.00	2.75	1.53	1.08	1.28	1.57	1.01	1.80	1.86	1.66

3.1 Single Observation Trigger

3.1.1 Method 1 (See Figure 3.2a):

1) Twelve baseline observations were collected, therefore n = 12.

2) The modified baseline observations were placed in sequential order from smallest to largest.

   [0.69, 0.84, 1.08, 1.10, 1.18, 1.26, 1.36, 1.54, 1.76, 2.11, 2.33, 4.54]

3) The number of observations, n, is less then 16, therefore the Single Observation Trigger (L) equals $x_{(12),}$ (the maximum) = 4.54.

4) All monitoring observations are less than 4.54, therefore the Single Observation Trigger (L) (4.54) was not exceeded.

3.1.2 Method 2 (See Figure 3.2b):

1) Twelve is an even number, therefore the median of the modified baseline observations is:

   $M = 0.5 * (x_{(6)} + x_{(7)}) = 1.31.$

   In order to determine M_1, calculate the median of the subset ranging from $x_{(7)}$ to $x_{(12)}$. Because 12 - 6 = 6 is even, $M_1 = 0.5 * (x_{(9)} + x_{(10)})$

   $M_1 = 0.5 * (1.76 + 2.11) = 1.935$

   Because M_1 is needed to calculate R, which must be based on unmodified concentrations, M_1 must also be calculated based on unmodified concentrations. The unmodified loads are ordered sequentially below:

   [0.69, 0.73, 1.08, 1.08, 1.10, 1.18, 1.36, 1.54, 1.62, 2.11, 2.33, 4.54]

   The median of unmodified baseline loads is:

   $M = 0.5 * (x_{(6)} + x_{(7)}) = 1.27.$

   The third quartile M_1 of the unmodified baseline loads is:

   $M_1 = 0.5 * (x_{(9)} + x_{(10)}) = 1.865.$

2) The first quartile M_{-1} of the unmodified baseline loads is:

   $M_{-1} = 0.5 * (x_{(3)} + x_{(4)}) = 1.08.$

3) Using the values of M_{-1} and M_1 calculated using the unmodified baseline loads,
$R = 1.865 - 1.08 = 0.785$.

4) $L = M_1 + (3 * R) = 1.935 + (3 * 0.785) = 4.29$

5) All monitoring observations are less than 4.29, therefore the Single Observation Trigger (L) was not exceeded.

3.2 Annual Comparison

3.2.1 Method 1 (See Figure 3.2a)

1) Twelve is an even number, therefore the median of the modified baseline observations is:
$M = 0.5 * (x_{(6)} + x_{(7)})$.
$M = 0.5 * (1.26 + 1.36) = 1.31$

The following steps are needed to calculate R. Therefore, the unmodified baseline loads must be used. These load observations are listed in listed from sequential order from smallest to largest:
[0.69, 0.73, 1.08, 1.08, 1.10, 1.18, 1.36, 1.54, 1.62, 2.11, 2.33, 4.54]

In order to determine M_1, calculate the median of the subset ranging from $x_{(7)}$ to $x_{(12)}$.
Because 12 - 6 = 6 is even, $M_1 = 0.5 * (x_{(9)} + x_{(10)})$
$M_1 = 0.5 * (1.62+ 2.11) = 1.865$

2) In order to determine M_{-1}, calculate the median of the subset ranging from $x_{(1)}$ to $x_{(6)}$.
Because 6 is even, $M_{-1} = 0.5 * (x_{(3)} + x_{(4)})$
$M_{-1} = 0.5 * (1.08 + 1.08) = 1.08$

3) To calculate R, subtract M_{-1} from M_1.
$R = 1.865 - 1.08 = 0.785$

4) The calculated value for R is then substituted into the equation for T.

$$T = 1.31 + \frac{1.815 * 0.785}{\sqrt{12}} = 1.72$$

5) The following monitoring observations are ordered from smallest to largest.
[1.01, 1.03, 1.08, 1.28, 1.53, 1.57, 1.66, 1.78, 1.80, 1.86, 2.00, 2.75]

6) There are 12 monitoring observations, therefore m = 12.
The number of observations is even, therefore $M' = 0.5 * (x_{(6)} + x_{(7)})$

$M' = 0.5 * (1.57 + 1.66) = 1.615$

This holds true for M_1' and M_{-1}' as well.

$M_1' = 0.5 * (x_{(9)} + x_{(10)}) = 0.5 * (1.80 + 1.86) = 1.83$

$M_{-1}' = 0.5 * (x_{(3)} + x_{(4)}) = 0.5 * (1.08 + 1.28) = 1.18$

7) To calculate R, subtract M_{-1}' from M_1'

$R' = 1.83 - 1.18 = 0.65$

8) The calculated value for R' is then substituted in the equation for T'.

$$T' = 1.615 - \frac{1.815 * 0.65}{\sqrt{12}} = 1.274$$

9) T ' (1.274) is less than T (1.72), therefore the median baseline pollution loading was not exceeded.

3.2.2 Method 2 (Wilcoxon-Mann-Whitney Test) (See Figure 3.2b)

1) When using both baseline and monitoring data, n = 12 and m =12

2) The modified baseline and monitoring observations are listed with their corresponding rankings.

Baseline Observations (lbs/day)	0.69	0.84	1.08	1.10	1.18	1.26	1.36	1.54	1.76	2.11	2.33	4.54
Baseline Rankings	1	2	*5.5*	7	8	9	11	13	16	21	22	24
Monitoring Observations (lbs/day)	1.01	1.03	1.08	1.28	1.53	1.57	1.66	1.78	1.80	1.86	2.00	2.75
Monitoring Rankings	3	4	*5.5*	10	12	14	15	17	18	19	20	23

Due to the fact that the value of 1.08 was obtained for two observations, an average ranking is used for this value. For 1.08, the average of 5 and 6 is 5.5.

3) The sum of the twelve baseline ranks (S_n) = 139.5.

4) From the table in section 1.2.2 of this appendix, the critical value (C) for 12 baseline and 12 monitoring observations is 99.

5) S_n (139.5) is greater than C (99). Therefore, according to the Wilcoxon-Mann-Whitney test, the monitoring observations did not exceed the baseline pollution loading.